Quantum non-triviality in two dimensional p-orbital Bose-Einstein condensates

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(Dated: December 13, 2012)

We investigate the quantum phase transition in a p-orbital bosonic system using Wilsonian renormalization group, where the p-orbital bosons condense at nonzero momenta and display rich phases including both time-reversal invariant and broken BEC states. We show that a quantum phase transition between the two different condensed states can be induced by the quantum fluctuations of the interactions between p_x and p_y orbital bosons even if the interactions are marginally irrelevant. This is a phenomenon beyond the well-known result of "quantum triviality" which states that in two dimensional boson systems the boson-boson repulsive interactions are marginally irrelevant so that the quantum critical behaviors of the systems are noninteracting, thus "trivial". This non-trivial phenomenon can be explained as an effect of the quantum fluctuation induced symmetry breaking. We also discuss the observation of these phenomena in the realistic experiment.

PACS numbers: 03.75.Nt, 05.30.Jp, 67.85.Jk, 67.85.Hj

Introduction.—Confining cold atoms in an optical lattice has proven to be an exciting and rich environment for studying many areas of physics [1–5]. However, the ground state wavefunctions of single component bosons are positive defined in the absence of rotation as described in the "no-node" theorem [6], which imposes strong constraint for the feasibility of using boson ground states to simulate many-body physics of interest. One way of circumventing this restriction is to consider the high orbital bands since the "no-node" theorem only applies to ground states [7]. The unconventional Bose-Einstein condensations (BECs) of high orbital bosons exhibit more intriguing properties than the ordinary BECs, including the nematic superfluidity [8, 9], orbital superfluidity with spontaneous time-reversal (TR) symmetry breaking [10–16] and other exotic properties [17–19]. The theoretic work on the p-orbital fermions is also exciting [20–25]. Furthermore, the p-orbital and multi-orbital superfluidity have been recently realized experimentally by pumping atoms into high orbital bands [3–5, 26, 27]. Hemmerich's group has successfully populated ^{87}Rb atoms into p-orbital bands in a double-well optical lattice with a life time of several tens of milliseconds [4]. The condensate momenta are located at half values of the reciprocal lattice vectors, which is consistent with the appearance of staggered ordering of orbital angular momenta.

Since the p-orbital Bose gas exhibits rich phase structures it is interesting to explore the quantum phase transition in such a system. In a general two dimensional boson system the problem of *Quantum triviality* exists. Quantum triviality was first discussed by L. D. Landau [28]. In quantum electrodynamics the perturbative renormalization group (RG) calculation shows that the renormalized charge vanishes as the cutoff sent to infinity. The theory is noninteracting, and thus "trivial". This indicates the failure of the perturbative theory. K. Wilson and J. Kogut also suggested that the ϕ^4 quantum field theory is in fact trivial in dimensions $d \geq 4$ [29] . Especially, at the upper-critical dimension $d = 4$ the triviality depends on whether the interaction is repulsive or attractive. If it's repulsive interaction, the renormalized coupling goes to zero as the length scale approaches infinity, therefore the theory is trivial. A paradigmatic description of dilute Bose gas is a ϕ^4 theory with dynamic exponent $z = 2$. At zero temperature the upper-critical dimension of such a system is $d = 2$. The renormalization group analysis shows that the boson-boson interaction is marginally irrelevant at one-loop level in two dimensions if the interaction is repulsive [30–32]. Hence, the s-orbital boson system is bound to quantum triviality in two dimensions. The p-orbital boson system is beyond the "no-node" paradigm, whose ground state wavefunctions can be complex-valued. It has richer interaction properties and quantum fluctuation effects than the s-orbital one, and hence may exhibit nontrivial behaviors beyond the quantum triviality.

In this Letter, we demonstrate that quantum triviality is genuinely nontrivial for a two dimensional p-orbital bosonic system in a square lattice. It turns out the interactions between the p_x and p_y orbital bosons can generate nontrivial effects even if they are marginally irrelevant. Before they renormalize to zero, these interactions can modify the flow of the chemical potentials and eventually change their flow directions, which lead to a quantum phase transition of the system. The spectrum of p-orbital bosons in square lattice has two energy minima in the Brillouin zone located at $\vec{K}_1 = (\frac{\pi}{a}, 0)$ for p_x-band and $\vec{K}_2 = (0, \frac{\pi}{a})$ for p_y-band respectively [4, 7]. At these two band minima the Bloch wave functions are time-reversal invariant and, thus, real valued. Lattice asymmetry favors a ground state at either $\vec{K}_1$ or $\vec{K}_2$, which is called real BECs. A linear superposition of these two real valued wave functions with a fixed phase difference forms a complex BEC, which is favored by the system with inter-

species interactions between p_x and p_y orbital bosons and spontaneously breaks the time-reversal symmetry [7, 14]. In our work we follow the spirit of "Shankar's RG" [33] to investigate if any quantum phase transition between the real and complex BEC phases can be induced when various interactions are turned on. Before proceeding to detailed calculation, we give a simple physical explanation of our result, which is illustrated in Fig. (1). We find that without inter-species interactions the complex BEC phase is confined in the first quadrant of the phase diagram as shown in (a) of Fig. (1). This represents a non-interacting system. Quantum triviality survives. However, as we turn on the inter-species interactions the complex phase is enlarged into the second and fourth quadrants as shown in (b), (c) and (d) of Fig. (1). That is, certain region of real BEC phases become unstable when the inter-species interactions are turned on and the system finally flows to the complex BEC phase. These interactions give rise to an instability from the real BEC phase to the complex BEC phase.

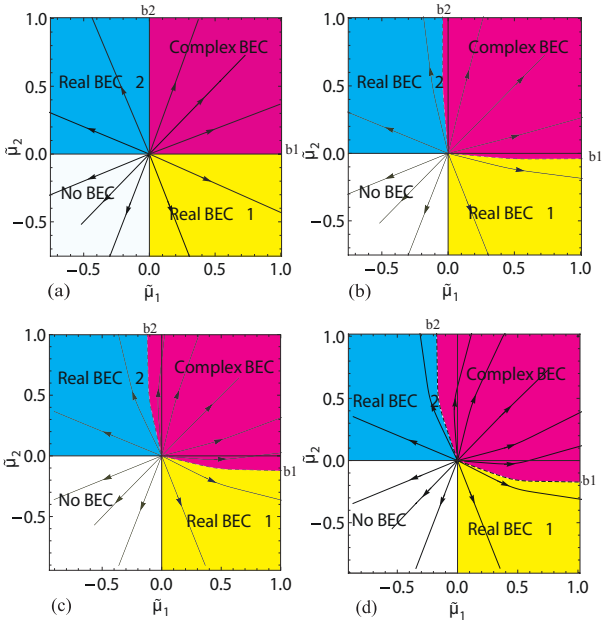

FIG. 1. (Color online) Phase diagrams of the p-orbital boson system. Four phases are determined by the flow directions of the chemical potentials $\tilde{\mu}_1$ and $\tilde{\mu}_2$ of p_x and p_y orbital bosons under the renormalization group transformation. In the complex BEC phase $\tilde{\mu}_1(\ell)$ and $\tilde{\mu}_2(\ell)$ run to positive infinities, which represents that both p_x and p_y orbital bosons condense. In the real BEC 1 (or 2) phase the p_x (or p_y) orbital boson condenses and the other one doesn't. In the no BEC phase neither of the two orbital bosons condense. The inter-species interaction coupling $\tilde{g}_3(0)$ between the p_x and p_y orbital bosons is set to $0, 0.1, 0.3$ and 0.5 for graphs (a), (b), (c) and (d) respectively. "b1" and "b2" denote boundary 1 and boundary 2 between the real and complex BEC phases.

The renormalization group flow equations and phase diagrams.—The tight-binding model of the p-orbital bosons in a square lattice shows that the energy minima are located at half values of the reciprocal lattice vectors [4, 7]. Around these two band minima $\vec{K}_1 = (\frac{\pi}{a}, 0)$ and $\vec{K}_2 = (0, \frac{\pi}{a})$ the system can be approximated as an interacting two-component Bose gas. The action of this low energy effective theory with the most general interactions is cast as following

$$\mathcal{S} = \mathcal{S}_1^0 + \mathcal{S}_2^0 + \mathcal{S}_1^I + \mathcal{S}_2^I + \mathcal{S}_{12}^I. \tag{1}$$

$\mathcal{S}_i^0 = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_0^{\Lambda} \frac{d^2k}{(2\pi)^2} \phi_i^*(\omega, \vec{k})(-i\omega + \epsilon_k - \mu_i)\phi_i(\omega, \vec{k})$ is the dynamic term of the boson field $\phi_i (i = 1, 2)$. Here we give a cutoff Λ to the momentum space since this is a low energy effective theory. μ_1 and μ_2 are chemical potentials of the low energy modes ϕ_1 and ϕ_2. The self-interaction term of each boson field is written as $\mathcal{S}_i^I = \int_{\omega k}^{\Lambda} g_i \phi_i^*(\omega_4, \vec{k}_4)\phi_i^*(\omega_3, \vec{k}_3)\phi_i(\omega_2, \vec{k}_2)\phi_i(\omega_1, \vec{k}_1)$. Here we used a short-handed notation $\int_{\omega k}^{\Lambda} = \prod_{i=1}^4 \int_{-\infty}^{\infty} \frac{d\omega_i}{2\pi} \int_0^{\Lambda} \frac{d^2k_i}{(2\pi)^2} (2\pi)^2 \delta(\vec{k}_4 + \vec{k}_3 - \vec{k}_2 - \vec{k}_1) \cdot (2\pi)\delta(\omega_4 + \omega_3 - \omega_2 - \omega_1)$. The most general inter-species interactions between ϕ_1 and ϕ_2 include two types of couplings $\mathcal{S}_{12}^I = \int_{\omega k}^{\Lambda} \Big\{ g_3 \phi_2^*(\omega_4, \vec{k}_4)\phi_1^*(\omega_3, \vec{k}_3)\phi_1(\omega_2, \vec{k}_2)\phi_2(\omega_1, \vec{k}_1) + g_4[\phi_1^*(\omega_4, \vec{k}_4)\phi_1^*(\omega_3, \vec{k}_3)\phi_2(\omega_2, \vec{k}_2)\phi_2(\omega_1, \vec{k}_1) + H.C.] \Big\}$. The g_4 term can rise in this high orbital model as an Umklapp scattering process since the momentum transfer is $\pm 2(\vec{K}_1 - \vec{K}_2)$, which equals to the reciprocal lattice vectors [7, 14].

In contrast to the mean-field theory where certain order parameter is presumably defined, the renormalization group analysis treats various instabilities on an equal footing without assuming any specific order parameters. We start from a microscopic theory and study the running of the couplings and chemical potentials using momentum-shell renormalization group method. Following the Wilson's approach [29] one finds that the chemical potentials scale according to the following relations up to one-loop order:

$$\begin{aligned} \frac{d\tilde{\mu}_1}{d\ell} &= 2\tilde{\mu}_1 + 4\tilde{g}_1 \cdot \theta(\tilde{\mu}_1 - 1/2) + \tilde{g}_3 \cdot \theta(\tilde{\mu}_2 - 1/2), \\ \frac{d\tilde{\mu}_2}{d\ell} &= 2\tilde{\mu}_2 + 4\tilde{g}_2 \cdot \theta(\tilde{\mu}_2 - 1/2) + \tilde{g}_3 \cdot \theta(\tilde{\mu}_1 - 1/2). \end{aligned} \tag{2}$$

In above equations we defined the dimensionless chemical potentials and coupling constants as $\tilde{\mu}_i = \mu_i m/\Lambda^2$ and $\tilde{g}_i = g_i m/(2\pi)$. The θ functions in the chemical potential flow equations are from ω integrations in the one loop calculations, where we have $\theta(\tilde{\mu} - 1/2) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \frac{e^{i\omega 0^+}}{i\omega - (1/2 - \tilde{\mu})}$. Notice that we introduced the factor $e^{i\omega 0^+}$ into the ω integral otherwise the integral over ω doesn't converge [33]. Most of the studies investigated the behaviors around the critical point, then these contributions can be ignored at zero temperature [31, 32]. However, in our case we focus on the asymptotic behaviors of the chemical potentials. The running behaviors of

μ_1 and μ_2 to $+\infty$ or $-\infty$ are used as the criteria of which phase the system will fall into. Therefore, we have to take these contributions into account. They will give severe influence to the running of the chemical potentials. The flow equations of the coupling constants are as following:

$$\frac{d\tilde{g}_1}{d\ell} = -2\tilde{g}_1^2 - 2\tilde{g}_4^2, \qquad \frac{d\tilde{g}_2}{d\ell} = -2\tilde{g}_2^2 - 2\tilde{g}_4^2,$$
$$\frac{d\tilde{g}_3}{d\ell} = -\tilde{g}_3^2, \qquad \frac{d\tilde{g}_4}{d\ell} = -2\tilde{g}_1\tilde{g}_4 - 2\tilde{g}_2\tilde{g}_4. \quad (3)$$

All the coupling constants with positive initial values are marginally irrelevant. For example, $\tilde{g}_3$ can be solved as $\tilde{g}_3(\ell) = \frac{\tilde{g}_3(0)}{1+\tilde{g}_3(0)\ell}$. It approaches to zero as the length scale ℓ goes to infinity. However, it doesn't imply that we can ignore these irrelevant coupling constants. This is because the small $\tilde{g}_i$ will generate small contributions to $\tilde{\mu}_i$, which will then quickly grow under the renormalization. As discussed by R. Shankar [33], an irrelevant operator can modify the flow of the relevant couplings before it renormalizes to zero.

The running chemical potentials $\tilde{\mu}_1(\ell)$ and $\tilde{\mu}_2(\ell)$ are relevant and can be solved numerically by Eq.(2). We find that in regions of $\tilde{\mu}_1(0) \geq \frac{1}{2}$ & $\tilde{\mu}_2(0) \geq \frac{1}{2}$ and $\tilde{\mu}_1(0) \leq 0$ & $\tilde{\mu}_2(0) \leq 0$, the running directions of $\tilde{\mu}_1(\ell)$ and $\tilde{\mu}_2(\ell)$ are completely determined by their initial values. If the initial values are positive or negative, they finally flow to positive or negative infinity. However, in other regions the running directions of the chemical potentials can eventually be changed by the one-loop corrections from the interaction couplings, even if they renormalize to zero. For instance, in Fig. (2) we start the running of $\tilde{\mu}_1(\ell)$ from a negative initial value. As we vary the interaction coupling $\tilde{g}_3(0)$ from 0.3 to 0.8 we observe that the running of $\tilde{\mu}_1(\ell)$ can finally be changed from the negative to the positive direction. That is, even if $\tilde{\mu}_1(\ell)$ runs to negative infinity at the tree level, the positive one-loop contributions can make $\tilde{\mu}_1(\ell)$ go to positive infinity eventually. The system will finally end up in a different phase.

Based on the numerical calculations the phase diagrams can be drawn in Fig. (1). The four phases are determined by the flow directions of $\tilde{\mu}_1(\ell)$ and $\tilde{\mu}_2(\ell)$ as $\ell \to \infty$. (I) Complex BEC: $\tilde{\mu}_1(\ell) \to +\infty$ and $\tilde{\mu}_2(\ell) \to +\infty$, (II) Real BEC 1 : $\tilde{\mu}_1(\ell) \to +\infty$ and $\tilde{\mu}_2(\ell) \to -\infty$, (III) Real BEC 2: $\tilde{\mu}_1(\ell) \to -\infty$ and $\tilde{\mu}_2(\ell) \to +\infty$ and (IV) No BEC: $\tilde{\mu}_1(\ell) \to -\infty$ and $\tilde{\mu}_2(\ell) \to -\infty$. In Fig. (1) the initial values of the couplings are $\tilde{g}_1(0) = \tilde{g}_2(0) = \tilde{g}_4(0) = 0.1$ for all of the four diagrams. $\tilde{g}_3(0)$ is given in the caption of Fig. (1). Comparing the four phase diagrams in Fig. (1), we see that as the inter-species interaction $\tilde{g}_3(0)$ becomes stronger the complex BEC phase get enhanced. Given that $\tilde{g}_3(0)$ is small, we can obtain the approximate expressions of the two boundaries. Boundary 1 is $\tilde{\mu}_2(0) = -\frac{\tilde{\mu}_1(0)\tilde{g}_3(0)}{1-\frac{1}{2}\ln(2\tilde{\mu}_1(0))\cdot\tilde{g}_3(0)}$ for $0 < \tilde{\mu}_1(0) < \frac{1}{2}$ and $\tilde{\mu}_2(0) = -\frac{\tilde{g}_3(0)}{2}$ for $\tilde{\mu}_1(0) > \frac{1}{2}$. Boundary 2 is

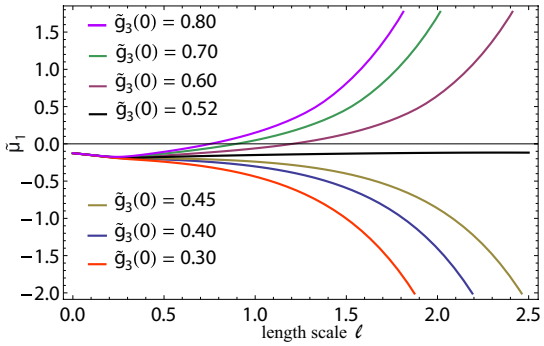

FIG. 2. (Color online) The flow directions of chemical potential $\tilde{\mu}_1(\ell)$ with different interaction coupling $\tilde{g}_3(0)$. As $\tilde{g}_3(0)$ is increased the running direction of $\tilde{\mu}_1(\ell)$ changes from negative infinity to positive infinity. The system can finally flow to a condensed phase. The initial values of the parameters are $\tilde{\mu}_1(0) = -0.125$, $\tilde{\mu}_2(0) = 0.3$ and $\tilde{g}_1(0) = \tilde{g}_2(0) = \tilde{g}_4(0) = 0.1$.

$\tilde{\mu}_1(0) = -\frac{\tilde{\mu}_2(0)\tilde{g}_3(0)}{1-\frac{1}{2}\ln(2\tilde{\mu}_2(0))\cdot\tilde{g}_3(0)}$ for $0 < \tilde{\mu}_2(0) < \frac{1}{2}$ and $\tilde{\mu}_1(0) = -\frac{\tilde{g}_3(0)}{2}$ for $\tilde{\mu}_2(0) > \frac{1}{2}$. They are indicated by "*b*1" and "*b*2" in Fig. (1).

Quantum fluctuation induced symmetry breaking (QFISB).—The mean-field results of the phase transition was derived by constructing a Ginzburg-Landau theory in Ref. [14]. However, starting from a microscopic theory our renormalization group analysis gives some qualitative differences: (I) In the mean-field analysis the boundary conditions between the real and complex BEC phases depend on the self-interaction couplings g_1 and g_2. However, these two couplings don't affect the phase boundaries in our results. The quantum phase transition is purely induced by the inter-species interaction between the p_x and p_y orbital bosons in RG analysis. (II) At one-loop level the g_4 term doesn't give any contributions to the flow equations of the chemical potential μ_1 and μ_2. It can get involved in higher order calculations. For instance, g_4 term can generate corrections to the boundaries "b1" and "b2" at two-loop level through the sunrise graphs in Fig. (3). However, in mean-field analysis the

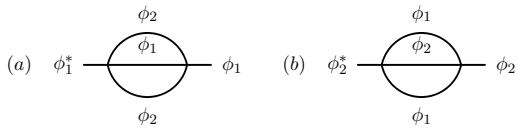

FIG. 3. The Feynman diagrams of the two-loop corrections to the chemical potential (a) μ_1 and (b) μ_2. ϕ_1 and ϕ_2 are the boson fields defined in Eq. (1).

phase boundaries depend on both g_4 and g_3. This difference originates from the starting points of the two analysis. The g_4 term explicitly breaks the original $U(1)\times U(1)$ symmetry to $U_D(1)$ symmetry, where the index "D" denotes for "Diagonal", and leads to a fixed phase difference between the two fields ϕ_1 and ϕ_2. The mean-field analy-

sis starts from this symmetry breaking phase. Hence, the complex phase in mean-field is a coherent superposition of the two ground states. However, our renormalization group analysis starts from the normal phase and focuses on the effects of the quantum fluctuations. In this case g_4 term doesn't show its contributions up to one-loop level. Our complex phase is just a incoherent mixture of the two ground states. (III) The comparison of the phase diagrams of the renormalization group analysis and mean-field theory can be illustrated in Fig. (4). In order to

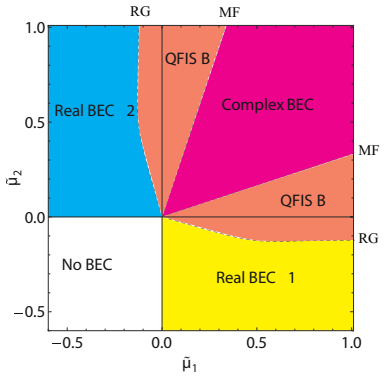

FIG. 4. (Color online) Comparison of phase boundaries from the renormalization group analysis and the mean-field analysis. "MF" and "RG" indicate the boundaries from the mean-field and renormalization group analysis respectively. The complex BEC phase in renormalization group analysis is larger than the one from mean-field theory by a region named "QFISB". The coupling constants are $\tilde{g}_1 = \tilde{g}_2 = 0.5$, $\tilde{g}_3 = 0.3$ and $\tilde{g}_4 = 0$.

compare the two phase diagrams with the same circumstance we set $g_4 = 0$ in the mean-field phase diagram. The expressions of the mean-field phase boundaries are $\frac{\tilde{\mu}_2}{\tilde{\mu}_1} = \frac{2\tilde{g}_2}{\tilde{g}_3}$ and $\frac{\tilde{\mu}_2}{\tilde{\mu}_1} = \frac{\tilde{g}_3}{2\tilde{g}_1}$ [14]. In Fig.(4) it's obvious to see that the crucial difference is that the complex BEC phase is enlarged and the real BEC phase is suppressed in the RG phase diagram.

In essence, the above differences can be explained as effects of the "quantum fluctuation induced symmetry breaking". The phase transition from the real BEC to complex BEC indicates the $U(1)$ symmetry breakdown of field ϕ_1 or ϕ_2. Mean-field description of the symmetry breaking is based on semiclassical approximation. In other words, it's a tree-level result. When we take into account the quantum fluctuation, the one-loop corrections can significantly change the model parameters and make some region of the real BEC phase become unstable. In Fig. (4) these regions are labeled by "QFISB". The possibility that the quantum fluctuations may produce spontaneous symmetry breaking was first discussed by S. Coleman and E. Weinberg [34]. They investigated a theory of a massless charged meson coupled to the electrodynamic field using effective potential method. Starting for a model without symmetry breaking at tree level they found that the one-loop effective potential indicated that a new energy minimum was developed away from the origin. This symmetry breakdown of the massless scalar field is completely induced by the quantum fluctuations of the electromagnetic field. Our theory reaches a qualitatively analogous result using renormalization group analysis. One boson field plays the role of the electromagnetic field and causes the symmetry breakdown of the other boson field through the interactions between them.

Experimental proposal.—We finally consider the experimental feasibility. One leading candidate to observe QFISB in p-orbital BEC is ^{87}Rb atoms in a bipartite optical square lattice [4]. The optical potential can be constructed by crossing two laser beams with wavelength $\lambda = 1,064$nm and $1/e^2$ radius $w_0 = 100\mu$m. The optical potential reads $-\frac{V_0}{4}e^{-\frac{2z^2}{w_0^2}}|\eta[(\hat{z}\cos\alpha + \hat{y}\sin\alpha)e^{ikx} + \epsilon\hat{z}e^{-ikx}] + e^{i\theta}\hat{z}(e^{iky} + \epsilon e^{-iky})|^2$. $\epsilon < 1$ and $\eta < 1$ describe the imperfect reflection and transmission efficiencies, respectively. The typical values of ϵ and η are $\epsilon \approx 0.81$ and $\eta \approx 0.95$. A BEC of 2×10^5 ^{87}Rb atoms (in the $F = 2$, $m_F = 2$ state) is produced in the optical trap. With V_0 set to $V_0/E_{\rm rec} = 6.2$ the excitation of p-orbital band can be obtained by ramping θ from 0.38π to 0.53π within 0.2ms. The initial values of chemical potentials μ_1 and μ_2 can be properly chosen by tuning α, which is the angle between the z axis and the linear polarization of the incident beam.

We choose $\alpha = 0$ to prepare the system in the "QFISB" region of Fig. (4). The interaction strengths g_i can be varied by changing the optical potential V_0 [1, 2]. With different interaction strengths the momentum spectra can be obtained by taking the absorption images. For small interactions sharp peaks arise at momenta $K_{(-,+)}$ and $K_{(+,-)}$ in the Brillouin zone, where $K_{(\pm,\pm)} \equiv 1/2(\pm 2\pi\hbar/\lambda, \pm 2\pi\hbar/\lambda)$ [4]. This indicates a real BEC phase. For strong interactions sharp peaks should appear at the four momenta $K_{(+,+)}$, $K_{(-,-)}$, $K_{(+,-)}$ and $K_{(-,+)}$, which shows the appearance of the complex BEC phase. This would be a clear evidence of a phase transition from the real BEC to the complex BEC when we enhance the interactions.

Conclusions.—In summary, we have investigated the quantum phase transition of the p-orbital boson gas in a two dimensional optical lattice using renormalization group method. We find that in contrast to the well-known "quantum triviality" shown by the s-orbital boson system the p-orbital one exhibits non-trivial behaviors. A phase transition from the real BEC phase to the complex BEC phase can be induced by the interactions between p_x and p_y orbital bosons. This non-trivial phenomenon is an effect of the quantum fluctuation induced symmetry breaking. Hence, the p-orbital boson condensates can be a counterexample of the quantum triviality exhibited by general two dimensional bosonic systems.

Acknowledgements—It's a pleasure to thank Congjun Wu for suggesting the problem to us. We would also like to thank Jiangping Hu for useful discussions. This work is supported by the NKBRSFC under grants Nos. 2012CB821400, 2011CB921502 and 2012CB821305 and NSFC under grants Nos. 1190024 and 10934010.

[1] D. Jaksch, C. Bruder, J.I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. **81**, 3108 (1998).
[2] M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, Nature **415**, 39 (2002).
[3] T. Müller, S. Fölling, A. Widera, and I. Bloch, Phys. Rev. Lett. **99**, 200405 (2007).
[4] G. Wirth, M. Ölschläger, and A. Hemmerich, Nature Phys. **7**, 147 (2011).
[5] P. Soltan-Panahi, D. Lühmann, J. Struck, P. Windpassinger, and K. Sengstock, Nature Phys. **8**, 71 (2012).
[6] R. P. Feynman, *Statistical Mechanics, A Set of Lectures* (Addison-Wesley Publishing Company, Boston, Massachusetts, 1972).
[7] C. Wu, Mod. Phys. Lett. B **23**, 1 (2009).
[8] A. Isacsson and S. M. Girvin, Phys. Rev. A **72**, 053604 (2005).
[9] C. Xu and M. P. A. Fisher, Phys. Rev. B **75**, 104428 (2007).
[10] W. V. Liu and C. Wu, Phys. Rev. A **74**, 013607 (2006).
[11] C. Wu, W. V. Liu, J. E. Moore, and S. Das Sarma, Phys. Rev. Lett. **97**, 190406 (2006).
[12] V. M. Stojanović, C. Wu, W. V. Liu, and S. Das Sarma, Phys. Rev. Lett. **101**, 125301 (2008).
[13] A. B. Kuklov, Phys. Rev. Lett. **97**, 110405 (2006).
[14] Z. Cai and C. Wu, Phys. Rev. A **84**, 033635 (2011).
[15] M. Lewenstein and W. V. Liu, Nature Phys. **7**, 101 (2011).
[16] J. P. Martikainen and J. Larson, Phys. Rev. A **86**, 023611 (2012).
[17] C. Wu, D. Bergman, L. Balents, and S. Das Sarma, Phys. Rev. Lett. **99**, 070401 (2007).
[18] V. W. Scarola and S. Das Sarma, Phys. Rev. Lett. **95**, 033003 (2005).
[19] X. Li, Z. Zhang and W. V. Liu, Phys. Rev. Lett. **108**, 175302 (2012).
[20] C. Wu, Phys. Rev. Lett. **101**, 186807 (2008).
[21] K. Wu and H. Zhai, Phys. Rev. B **77**, 174431 (2008).
[22] E. Zhao and W. V. Liu, Phys. Rev. Lett. **100**, 160403 (2008).
[23] S. Zhang, H. H. Hung, and C. Wu, Phys. Rev. A **82**, 053618 (2010).
[24] X. Li, E. Zhao, and W. V. Liu, Phys. Rev. A **83**, 063626 (2011).
[25] M. Zhang, H. H. Hung, C. Zhang, and C. Wu, Phys. Rev. A **83**, 023615 (2011).
[26] J. Sebby-Strabley, M. Anderlini, P. S. Jessen, and J. V. Porto, Phys. Rev. A **73**, 033605 (2006).
[27] M. Ölschläger, G. Wirth, and A. Hemmerich, Phys. Rev. Lett. **106**, 015302 (2011).
[28] L. D. Landau, A. Abrikosov, and L. Khalatnikov, Nuovo Cim. Suppl. **3**, 80 (1956).
[29] K. G. Wilson and J. B. Kogut, Phys. Rep. **12**, 75 (1974).
[30] D. S. Fisher and P. C. Hohenberg, Phys. Rev. B **37**, 4936 (1988).
[31] S. Sachdev, T. Senthil, and R. Shankar, Phys. Rev. B **50**, 258 (1994).
[32] S. Sachdev and E. R. Dunkel, Phys. Rev. B **73**, 085116 (2006).
[33] R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).
[34] S. Coleman and E. Weinberg, Phys. Rev. D **7**, 1888 (1973).